The good, the bad, and the asymmetric: evidence from a conditional-density model

An asymmetrical opposite-signed-shocks GARCH model

Dmitry I. Malakhov, Higher School of Economics
(dmalakhov@hse.ru)

Andreï V. Kostyrka, University of Luxembourg
(andrei.kostyrka@uni.lu)

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Presentation outline

- Model specification
- Comparing our models to competing ones
    - Our models have lower AIC criteria and better VaR and volatility forecasts
- Interpretation of the model in the context of return structure
    - Volatility is insensitive to positive shocks (and slightly decreases after strong positive shocks), but increases drastically after strong negative ones
    - Skewness is negative in general and becomes even more pronounced (in a non-linear way) after strong negative shocks
- Extension: jumps (ongoing work)

Basic model

- Simple observation: agents react differently to positive and negative news.
- An asymmetric approach (analysis of 'bad' and 'good' variance) becomes a prominent area of research: Barndorff-Nielsen et al, 2008; Patton, Sheppard, 2015; Segal et al, 2015; Kilic, Shaliastovich, 2018.
- Log-returns are decomposed into a constant mean, 'good' shocks and 'bad' shocks:

$$r_t = \mu + \varepsilon_t^+ + \varepsilon_t^-, \quad \varepsilon_t^+ = \sqrt{\sigma^2_{\varepsilon_t^+}} \cdot e_t^+, \quad \varepsilon_t^- = \sqrt{\sigma^2_{\varepsilon_t^-}} \cdot e_t^-.$$

  where e_t^+ are IID 'good' baseline shocks, e_t^- are IID 'bad' baseline shocks, both from a shape-scale family.

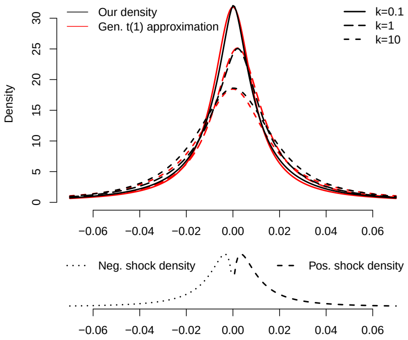
Distributions and copulæ

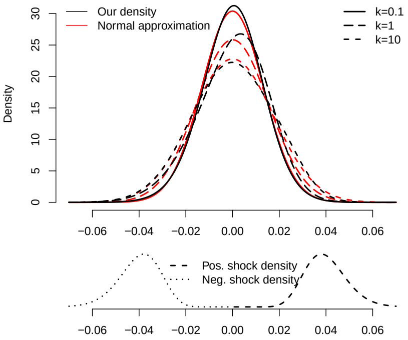
- Case 1: Non-zero-mean shocks: $\operatorname{supp} e_t^+ = [0, +\infty)$, $\operatorname{supp} e_t^- = (-\infty, 0]$. Case 2: Zero-mean shocks: $\operatorname{supp} e_t^+ = [\underline{e^+}, +\infty)$, $\operatorname{supp} e_t^- = (-\infty, \overline{e^-}]$.
- ε_t^+ and ε_t^- are copula-connected with a dynamic parameter:

$$F_{\varepsilon_t^+, \varepsilon_t^-}(x, y) = S\big(F_{\varepsilon_t^+}(x), F_{\varepsilon_t^-}(y); \kappa_t \;\big|\; \Omega_{t-1}\big), \quad \kappa_t = g(\kappa_{t-1}, r_{t-1})$$

- Shock distributions: gamma, centred gamma, log-logistic, centred log-logistic.
- Copulæ: independence, Plackett, cubic, AMH, Clayton.
- Subsumes Bekaert et al. (2015) as a special case

Examples of densities from our model





Left: the shocks are log-logistic with shapes $\theta^+ = \theta^- = 1.5$, the conditional scales are $\sigma_{\varepsilon_t^+} = \sigma_{\varepsilon_t^-} = 0.01$.

Right: the shocks are gamma-distributed with sh apes $\theta^+ = \theta^- = 20$, the conditional scales are $\sigma_{\varepsilon_t^+} = \sigma_{\varepsilon_t^-} = 0.002$.

Parameter dynamics

- The positive and negative scale parameters have a GJR-GARCH specification:

$$\begin{cases} \sigma^2_{\varepsilon_t^+} = \alpha_0 + \alpha_1 \sigma^2_{\varepsilon_{t-1}^+} + \alpha_2 \tilde{r}^2_{t-1} + \alpha_2^- \mathbb{I}_{\tilde{r}_{t-1}<0} \tilde{r}^2_{t-1}, \\ \sigma^2_{\varepsilon_t^-} = \beta_0 + \beta_1 \sigma^2_{\varepsilon_{t-1}^-} + \beta_2 \tilde{r}^2_{t-1} + \beta_2^- \mathbb{I}_{\tilde{r}_{t-1}<0} \tilde{r}^2_{t-1}. \end{cases}$$

- We try $\tilde{r}_t \stackrel{\text{def}}{=} r_t - \mathbb{E}_{t-1} r_t$ (de-meaned) or $\tilde{r}_t \stackrel{\text{def}}{=} r_t$ (no de-meaning). We report results for models with **de-meaned** $\tilde{r}_t$ only (no substantial differences).
- Copula parameter specification ($p \in \{0, 2, 3\}$):

$$\kappa_t = \gamma_0 + \mathbb{I}_{p\in\{2,3\}} [\gamma_1 \kappa_{t-1} + \gamma_2 \tilde{r}^p_{t-1} + \gamma_2^- \mathbb{I}_{\tilde{r}_{t-1}<0} \tilde{r}^p_{t-1}].$$

- Dynamic scale is more appealing, but we also replicate Bekaert et al. (2015) with dynamic shape.

Conditional density function

- Conditional distributions are implied: $F_{\varepsilon_t^+,\varepsilon_t^-}(x,y) \equiv F_{\varepsilon_t^+,\varepsilon_t^-}(x,y \mid \Omega_{t-1})$.
- Joint density function of shocks:

$$
\begin{aligned}
f_{\varepsilon_t^+,\varepsilon_t^-}(x,y) &= \frac{\partial^2}{\partial x \partial y} F_{\varepsilon_t^+,\varepsilon_t^-}(x,y) \\
&= f_{\varepsilon_t^+}(x) \cdot f_{\varepsilon_t^-}(y) \cdot s\big(F_{\varepsilon_t^+}(x), F_{\varepsilon_t^-}(y)\big),
\end{aligned}
$$

  where s is the cross-derivative of the copula function.
- Given the joint density, we obtain the PDF of the sum of shocks $\psi_t \stackrel{\text{def}}{=} \varepsilon_t^+ + \varepsilon_t^-$:

$$
f_{\psi_t}(z) = \int_{\operatorname{supp} \varepsilon_t^-} \Big[f_{\varepsilon_t^+}(z-v) \cdot f_{\varepsilon_t^-}(v) \cdot s\big(F_{\varepsilon_t^+}(z-v), F_{\varepsilon_t^-}(v)\big)\Big] \, \mathrm{d}v.
$$

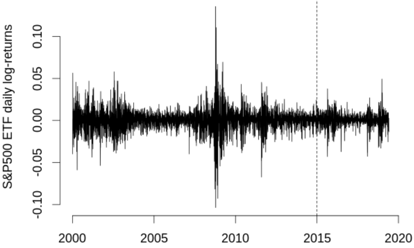
- Maximum likelihood estimation done with BFGS from a candidate point found via a stochastic search

Performance of our models vs competing ones

- We benchmark our models against all relevant well-established $\ldots$ARCH(1, 1) models.
- 40 variants (more than in Hansen & Lunde (2005)):
  - 10 volatility dynamics: GARCH, eGARCH, GJR-GARCH, TGARCH, apARCH (asymmetric power), cGARCH (component), AVGARCH (absolute value), NGARCH (non-linear GARCH), NAGARCH (non-linear asymmetric), family GARCH;
  - 4 distributions of shocks: normal, skewed normal, Student, skewed Student.
- All models were reëstimated every 2 points, and a rolling-window 1-step VaR and annualised volatility forecasts were obtained.

Data

- Daily returns of ETF on S&P500 from 01/01/2000 to 31/05/2019.
- Estimation period: 15 years (3773 points).
- Testing period: 4.5 years (1110 points).



Tests for model comparison

1. Christoffersen (1998) conditional coverage (CC) backtesting
   - $\mathcal{H}_0:$ exceedances are independent, and their proportion is correct.
2. Engle—Manganelli (2004) dynamic quantile (DQ).
   - $\mathcal{H}_0:$ exceedances do not depend on their past values, and their proportion is correct.
3. Information criterion: AIC.

The paper contains more tests.

Results: VaR out-of-sample test details

Distr	Cop	p	VR_{out}	p_{CC}	p_{DQ}
c-log-log	Clayton	3	.96	.63	.23
log-log	Clayton	3	.85	.08	.19
c-log-log	Plackett	3	.85	.20	.06
c-log-log	Plackett	0	.82	.11	.14
c-log-log	Plackett	2	.82	.11	.09
gamma	Plackett	0	.78	.13	.35
gamma	Plackett	3	.78	.13	.29
BEGE*	none	0	.67	.00	.01

·ARCH	Distr	VR_{out}	p_{CC}	p_{DQ}
csGARCH	s-$\mathcal{N}$	.85	.26	.06
gjrGARCH	s-t	.82	.10	.11
apARCH	s-$\mathcal{N}$	.78	.33	.12
apARCH	s-t	.76	.26	.07
TGARCH	s-t	.75	.26	.07
ALLGARCH	s-$\mathcal{N}$	.73	.39	.20
TGARCH	s-$\mathcal{N}$	.73	.36	.12
eGARCH	s-t	.73	.35	.40

* Bekaert et al. (2015): dynamic shape, centred gamma shocks, centred returns, no copula

Distributions. c-: centred, s-: skewed; log-log: log-logistic.

p: power of $\tilde{r}$ in the dynamics of copula parameter

VR: Violation ratio (observed/expected) out of sample

CC: Christoffersen's Conditional Coverage test p value

DQ: Engle—Manganelli's Dynamic Quantile test p value

Green denotes good values of statistics, red indicates bad values.

Results: Log-likelihood and AIC

Model	Log-lik*.	AIC*
c-log-log Clayton 3	147.04	−264.09
c-gamma AMH 3	145.29	−260.58
Bekaert et al. (2015) BEGE	133.39	−244.78
log-log cubic 2	134.44	−238.89
log-log cubic 3	133.06	−236.12
c-log-log Plackett 0	129.37	−234.74
c-log-log Plackett 3	131.83	−233.67
c-log-log Plackett 2	131.79	−233.57
AVGARCH skew-t	121.79	−227.58
TGARCH skew-t	114.69	−215.38
eGARCH skew-t	113.90	−213.80
apARCH skew-t	114.70	−213.41
NAGARCH skew-$\mathcal{N}$	101.81	−191.62
ALLGARCH skew-$\mathcal{N}$	102.94	−189.88
gjrGARCH skew-t	96.88	−179.76
AVGARCH skew-$\mathcal{N}$	87.94	−161.88

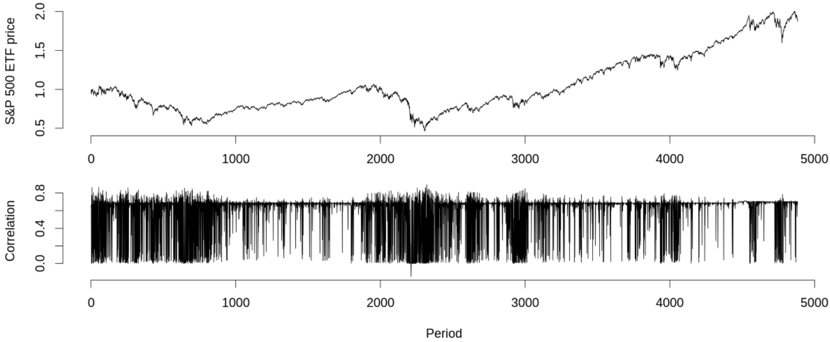
Values of log-lik. minus 12 000 and AIC plus 24 000 are shown. Green denotes good values of statistics, red indicates bad values.

Results: Inference

Term	Estimate	t-stat	p	QML t-stat	QML p
Const+	$-1.16 \cdot 10^{-6}$	-0.3	0.761	-0.25	0.805
GARCH+	0.978	> 100	0	> 100	0
ARCH+	-0.205	-0.68	0.499	-1.54	0.123
ARCH+ $\cdot \mathbb{I}_{\tilde{r}_{t-1}<0}$	1.26	0.85	0.393	0.67	0.504
Const−	$3.72 \cdot 10^{-5}$	1.75	0.081	0.8	0.422
GARCH−	0.812	> 100	0	> 100	0
ARCH−	-0.725	-12.38	0	-4.89	0
ARCH− $\cdot \mathbb{I}_{\tilde{r}_{t-1}<0}$	7.48	23.83	0	36.78	0
Cop. const.	3.34	1.68	0.093	1.59	0.111
Cop. GARCH	-0.310	-12.05	0	-28.36	0
Cop. ARCH	$-1.34 \cdot 10^{6}$	-1.82	0.069	-1.58	0.114
Cop. ARCH $\cdot \mathbb{I}_{\tilde{r}_{t-1}<0}$	$3.20 \cdot 10^{6}$	1.82	0.068	1.56	0.118
Shape+	15.83	2.00	0.046	1.24	0.214
Shape−	11.75	11.63	0	24.83	0
μ	$9.38 \cdot 10^{-5}$	0.73	0.468	0.62	0.538

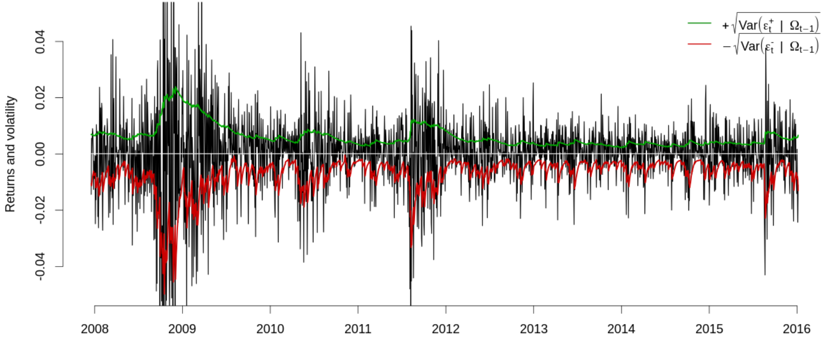
Dynamic correlation

Model: dynamic scale, centred log-logistic distribution, Clayton copula, cubes of de-meaned return in dynamics



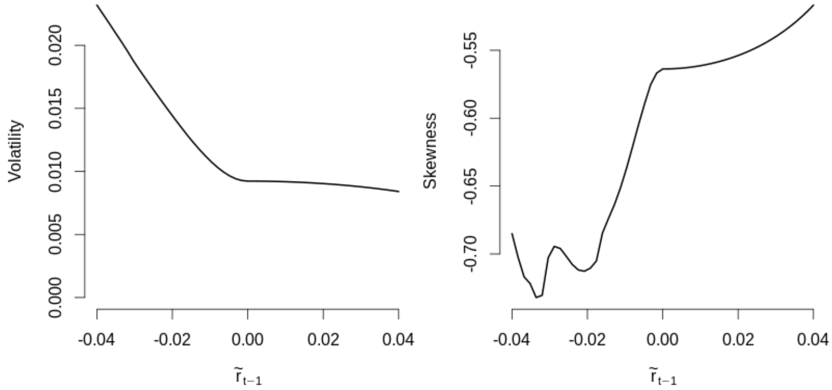
'Good' and 'bad' dynamic volatility

Model: dynamic scale, centred log-logistic distribution, Clayton copula, cubes of de-meaned return in dynamics



News impact curves for volatility and skewness

Model: dynamic scale, centred log-logistic distribution, Clayton copula, cubes of de-meaned return in dynamics



Extension: model with jumps

Consider having both continuous and discrete jumps:

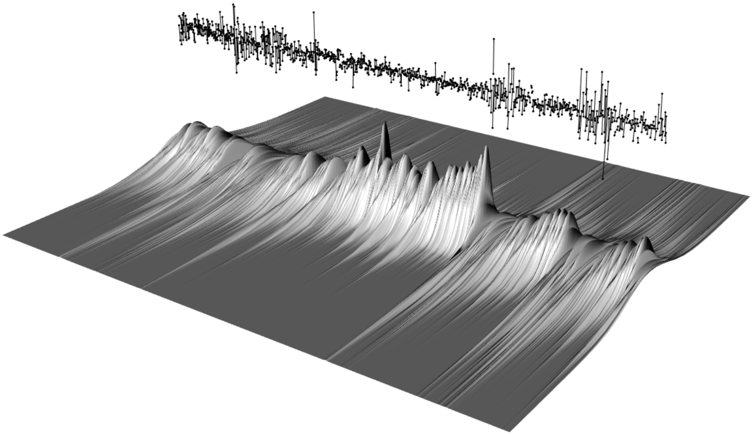
$$r_t = \mu + \varepsilon_t^+ + \varepsilon_t^- + \nu_t^+ + \nu_t^-,$$

- $\varepsilon_t^+, \varepsilon_t^-$ are **continuous** shocks, ν_t^+, ν_t^- are **discrete** shocks. The number of discrete shocks is a Poisson random variable with dynamic intensities λ_t^+, λ_t^-.
- There are three copulæ describing their joint distribution: $C\big(S(F_{\varepsilon^+}, F_{\varepsilon^-}), J(F_{\nu_t^+, \nu_t^-}))$
- Double and triple integrals are evaluated to obtain the density of r_t (computationally intensive!).
- Preliminary result: the average jump size is negative.

Conclusions

- Our model accounts for many sources of asymmetry in the distribution of returns, including correlation between unobserved shocks.
- A model with heavy-tailed centred unobserved shocks with a dynamic copula is the best one for out-of sample risk measure forecasting.
- It helps reveal the structure of returns: ‘good’ volatility is persistent, and only ‘bad’ volatility increases due to negative shocks.
- Correlation between ‘good’ and ‘bad’ shocks is positive most of the time and close to zero during the market turmoil.

Thank you for your attention!



Mean absolute error for OOS volatility forecast

Model	MAE
c-log Plack. 0	4.23
c-log Clayt. 3	4.24
c-log Plack. 3	4.25
TGARCH skew-$\mathcal{N}$	4.29
apARCH skew-$\mathcal{N}$	4.30
logl Plack. 3	4.30
ALLGARCH skew-$\mathcal{N}$	4.31
eGARCH skew-$\mathcal{N}$	4.33
TGARCH skew-t	4.35
apARCH skew-t	4.35
gamm Plack. 3	4.37
c-log Plack. 2	4.37
AVGARCH skew-$\mathcal{N}$	4.38
NAGARCH skew-$\mathcal{N}$	4.38
gamm Plack. 2	4.39
logl AMH 0	4.41

Bekaert et al. (2015)-like model quality

Copula	p	VR^{i}	p^{i}_{CC}	p^{i}_{DQ}	$p^{\text{i}}_{\text{dur}}$	AIC*	VR^{o}	p^{o}_{CC}	p^{o}_{DQ}	$p^{\text{o}}_{\text{dur}}$
—	—	0.973	0.054	0.292	0.001	−244.8	0.673	0.003	0.011	0.411
Plackett	0	1.000	0.000	0.266	0.007	−226.7	0.691	0.005	0.019	0.441
Plackett	2	1.043	0.796	0.271	0.014	−274.8	0.782	0.018	0.036	0.282
Plackett	3	1.043	0.796	0.408	0.007	−275.2	0.782	0.018	0.041	0.368
cubic	0	0.883	0.012	0.175	0.001	−244.1	0.709	0.009	0.031	0.551
cubic	2	0.915	0.016	0.166	0.002	−218.2	0.618	0.000	0.002	0.358
cubic	3	0.984	0.050	0.099	0.000	−274.5	0.727	0.015	0.048	0.596
AMH	0	0.995	0.031	0.122	0.002	−222.9	0.691	0.005	0.005	0.356
AMH	2	0.995	0.200	0.740	0.001	−230.3	0.709	0.009	0.028	0.551
AMH	3	0.968	0.309	0.781	0.008	−228.4	0.709	0.009	0.030	0.551
Clayton	0	1.043	0.146	0.232	0.003	−208.1	0.636	0.001	0.001	0.176
Clayton	2	1.000	0.000	0.748	0.001	−236.0	0.673	0.003	0.002	0.263
Clayton	3	0.952	0.155	0.353	0.000	−213.0	0.600	0.001	0.014	0.519

Centred gamma distribution. de-meaned $\tilde{r}_t$. $^{\text{i}}$ for in-sample performance, $^{\text{o}}$ for out-of-sample. p: the power used for returns in the copula dynamics. VR: Violation ratio.

p_{CC}, p_{DQ}, p_{dur}: p value of the conditional coverage, dynamic quantile, and no-hit duration test. AIC*: Akaike information criterion plus 24 000.